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ABSTRACT

This report details a visit to China by a delegation from the Illinois Council of Teachers of Mathematics. The 3-week visit included 14 schools in 8 cities. The report includes general impressions of the society, a description of the general educational system and a more detailed look at mathematics programs. The report discusses mathematics curricula at the primary and middle schools, as well as mathematics teacher preparation. The summary and conclusions section includes the following observations: (1) Chinese students try harder; (2) Chinese students spend more time in mathematics classes; (3) parents, teachers, and society in general expect more; and (4) Chinese teachers feel more positive about their jobs and their role in society. (PK)

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Primary and Secondary Mathematics Education in China: Report of the ICTM China Mathematics Delegation (1987)

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"Mathematics Is Basic To The Study Of All Other Subjects" -
a middle school principal

"Smart Students Work Hard And Learn More" -
Hua Logeng, China's most important mathematical
figure of the last 40 years

"Study, Study, Study Some More"

"Work Diligently For The Future Of The Motherland"

"Unite And Make Progress"

Posters written in Chinese characters which we
observed in mathematics classrooms.

Some General Observations and Impressions

- China's population is now more than 1,100,000,000 - nearly five times that of the United States.
- There are nearly as many students in Chinese primary and secondary schools as there are people in the United States.
- There exists a rigorous campaign to limit family size to one child.
- The priorities in China's development are the Four Modernizations: Industry, Science and Technology, Agriculture, and National Defense.
- Mathematics holds a long and important tradition in Chinese history.
- China has a National Teacher's Day, honoring all school teachers.
- Parents, teachers, administrators and peers have high expectations of students in school.
- There is a unified mathematics curriculum, called the New Unified Series, used throughout all of China.
- In some primary schools, teachers stay with the same class for 2-4 years.
- In moral education classes, students are taught how to behave.
- Chinese students attend school about 240 days per year, 6 days a week for 40 weeks.

- All students take mathematics each year they are in school.
- The mathematics curriculum is intensive.
- Mathematics is often taught during the first two periods of the day along with Chinese Language.
- Teachers frequently have only 2 periods of instruction (out of six) each day.
- Students stand and greet their teachers at the beginning of the class, and stand and say goodbye at the end.
- Class sizes range from 40-60 students.
- Eye exercises are done by all students each day (designed to prevent myopia).
- The teaching of geometry and algebra begins in the primary school.
- Entrance examinations are required of students to enter grade 7 and grade 10 of the key middle schools and all universities.
- Homework is required which involves a small number of problems, significant problem solving, and is checked regularly.
- Note-taking during class instruction is not encouraged - students are expected to observe the teacher and answer questions.
- There is a growing use of microcomputers in Chinese education.
- Mental arithmetic and problem solving are an important part of the primary school curriculum.
- A Chinese student who has finished compulsory education (9 years) has already studied most of what we consider high school geometry and algebra.
- There is still wide-spread use of the Chinese abacus (Suanpan) by merchants/business people.

Background

The China visit was sponsored by the Illinois Council of Teachers of Mathematics (ICTM). The delegation was formed following an invitation from the Chinese Mathematical Society, whose members come from the ranks of mathematicians and experienced teachers from all school levels. Arrangements from the United States with the China International Conference Center For

Science and Technology (CICCST) in Beijing were handled by the US - China People's Friendship Association (USCPFA) in Washington, D.C. All arrangements in China were handled by the China Association For Science and Technology (CAST) in Beijing and its branches in the cities/provinces visited by the delegation. International travel was handled by B and A Travel in Carbondale, Illinois which made a contribution of $2000 to ICTM's Scholarship Fund.

The delegation included teachers, administrators and Educational Service Center personnel from all levels (elementary through university) throughout Illinois. In total, members of the delegation were experienced with virtually all aspects of mathematics teaching. The group visited a total of fourteen schools in eight cities (Beijing, Xian, Nanjing, Wuxi, Hangzhou, Shanghai, Guilin, and Guangzhou [Canton]). It also visited or had discussions with staff in other places such as a bookstore, a Children's Palace, People's Publishing House, a math teachers home, and school policy-making groups.

At the top of the delegation's priorities were visits to classrooms and discussions with teachers and students. Not only was there a healthy exchange of ideas and practices centering on the professional life of teachers, but there was also an exchange of materials. Various American publishers/ manufacturers (see acknowledgements) donated a variety of books, pamphlets, monographs, teaching aids, and calculators which were given to Chinese counterparts who, in turn, presented a variety of materials to our delegation. The delegation was also invited to four magnificent banquets hosted by outstanding Chinese scholars and leading members of CAST.

The delegation had periodic, informal discussions of what we were observing in classrooms during the March 1-21, 1987 visit and a "summing up" session in Hong Kong. The delegation reconvened six weeks later in Decatur, Illinois to discuss the visit and prepare this report. Since change is the

one thing that seems to remain constant in China and because China is undergoing such rapid change, the delegation wants its observations added to those of other groups, such as the 1977 NCTM delegation (Madell & Becker, 1979; Swetz, 1979), the 1980 and 1984 delegations of the US - China People's Friendship Association (Ware & Becker, 1983; Becker, 1984) and the People-to-People delegation (Meserve, Lichtenberg, & Leitzel, 1984; Steen [Ed.], 1984).

Rhythm of Life

It is natural when visiting a foreign country to observe the people and, as much as possible, what is "going on." Those who visit China are quickly immersed in the movement of people. As we travelled by air, train, bus, cars, boat, and foot we observed people moving about by foot, bicycles, oxen- and horse-drawn wagons, cars, trucks, buses, motorcycles, and walking tractors, all on the same thoroughfares! It was common for us to feel impending disaster in this huge movement, but we saw only two accidents. For vehicles, we concluded that the two most used parts were the horn (bells on bicycles) and gas pedal. Small two-wheel motor driven "walking tractors" are seen everywhere in China - in and out of the cities. They pull trailers and just about anything.

Bicycles. Bicycles are everywhere - millions of them - in both urban and rural settings. We saw parking lots for bicycles everywhere - park it, lock it, and leave it. Bicycles are simple, but sturdily built one-speeds. Frequently, a bicycle has two or three riders - the driver, a child in a small seat, or a spouse or friend on the back. But bicycles are also great freight carriers - large bundles of produce on either side of the back wheel and piled high above it. Three wheelers might be carrying a sofa (with or without a person on it), beds, refrigerators, etc. We could not help admiring the pedalers physical stamina.

Physical Exercise. Chinese of all ages exercise regularly. China "comes to life" early, by 5:00 a.m. Walk anywhere in a city and you see people jogging, doing Tai Chi, calesthenics, or other exercise forms, either individually or in groups. In the rural areas one sees farmers walking behind water buffalos plowing rice or wheat patties or digging small irrigation ditches. If people rise early, so they also retire early. Streets are bare by 8 p.m. except, perhaps, when movies let out by 9-10 p.m. and even then people quickly go home.

Friendliness. Friendly people are encountered from the moment one arrives in China. Smile and total strangers smile back. We were frequently approached on streets by people who wanted to practice their English. No sooner does a conversation begin than one finds a crowd gathered to observe the goings-on. Or if we were in a shop or department store making a purchase, a helpful person approached to assist, followed by a whole audience of naturally curious people. Not being embarrassed in the least, they would frequently move very close (touching or crowding) to watch and listen to the transaction. Commonly we were referred to as foreign guests.

Open Doors and Windows. Even when it is cold, doors and windows are open. It was chilly in several cities we visited and after several days we learned to dress accordingly - by layered clothing as is common among Chinese. Aside from hotels and regardless of the nature of our activity (professional or otherwise), most buildings we were in had no heating. But life went on in a natural way. Just as there is no central heating, so there is no central air-conditioning, except in hotels for tourists.

Food. We were treated to scrumptious meals. We feasted on a multitude of meat, fish, and vegetable dishes, steamed buns, rice, bean curd, fruit, mushrooms, lotus root, nuts, and a host of other really fine foods and

desserts. Of course, the Chinese do not eat like this except on special occasions and even then they only take a little of each dish, a practice which many of us eventually picked up. The Chinese practice is to have a long lunch hour which permits a rest between morning and afternoon activity. We used this time to make other visits to such places as a bookstore, handicrafts factory, site of historical/cultural interest, or just to start out early to a professional visit.

Children and Senior Citizens. It is obvious, even to the casual observer, that the Chinese reserve special consideration for both the very young and the old. Senior citizens are accorded special respect and deference. Grandparents, along with parents, play an important role in bringing up children. It is common to see grandparents with children at the markets in the morning, or meeting children after school to escort them home. While parents are at work, grandparents often care for the children. The Chinese have an extensive system of child care centers, nursery schools, and kindergartens (for children ages 3-5) which are connected to factories, schools, universities, and various other institutions. The general attitude is to provide warm and loving care for children who are openly regarded as China's future.

Themes in Chinese Society and Education

Just as it is common to observe what we have loosely termed the rhythm of life, so it is natural to look for general themes which pervade Chinese society and education. During and immediately following the Cultural Revolution, Socialist-Egalitarianism was the principal Maoist value in education. The main objective was to ward off elitism. One manifestation of this was the practice of having teachers, professors, students and other professional workers engaging in physical labor by working in industrial

plants, school and university run factories or on farms. But there is much less obvious emphasis on this now even though a work-study program remains an important policy in Chinese education.

Serve the People is another major theme. Pride is instilled in primary and middle (secondary) school students in helping others and individual preferences are not expressed. Orienting education towards social utility and an attitude of everybody being an equal contributor to the national effort are still characteristics of Chinese society. What we might term the collective is still a striking characteristic of Chinese life. A student is considered part of a group and people, in general, have responsibility for others. When someone has difficulty or gets "out of line," it is the responsibility of the group to help that person out - the welfare of the group depends on all members of the group. This omnipresent pressure to conform can be seen in group activities in schools. However, there have been some changes in this with the establishment of key schools, mathematics contests and admission tests for middle schools and universities.

During the period of the 1960's-70's, Self Reliance was a major theme in society. For example, national development free from foreign influence was very common. However, one sees a large divergence from this policy now with China reaching out and adopting an open-door policy in which foreign investment, joint ventures, and expertise are evident in all parts of China. One sees this in education as well as other aspects of Chinese national life.

An intense Patriotism is instilled in children from pre-school and kindergarten onwards through the middle schools. This was a major theme of Chinese life in 1949 and is unchanged today. An example of this is the revolutionary music played over the public address system while students engage in eye exercises and physical culture every morning.

Many changes which have recently occurred in China are due to wide ranging changes in national policy. For example, great emphasis is placed on Science and Technology in China. Following the Cultural Revolution in 1978, the government convened a National Science Conference in Beijing to lay out national policy for scientific development. There followed, a month later, a National Work Conference on Education and by Fall, 1978, a national group of mathematics educators met in Beijing to recommend details and set in motion large scale curriculum development in mathematics. Within a year and a half a New Unified Series of mathematics textbooks for primary and middle schools were being used in all schools throughout China. This new curriculum went through a subsequent revision and is presently under major revision again.

Reference was made to the Four Modernizations earlier. Based on background reading before going to China and observations during our visit, we are tempted to label Education as a fifth modernization. Contrary to the emphasis on political attitudes (or "redness" as it was called) during the Cultural Revolution, the emphasis is now clearly on academics or "expertness." These two concepts have competed in China since Liberation in 1949 and have resulted in periods of turmoil. How well the two concepts continue to harmonize in practice is one of the most interesting phenomena to watch in China during the late 80's and 90's. A level of harmony will in all probability lead towards continued progress in China's efforts toward modernization.

Structure of Education

Many students, especially in urban areas, begin their formal education as early as age 3 in pre-schools and kindergartens. Primary education is being restructured to include the following types of schools: countryside (simple) and countryside (adult) - both designed specifically to eliminate illiteracy; "regular" primary (5 years) in rural areas; primary (6 years) in urban areas.

If the resources available to the government are scarce, as they are, then they are also distributed unevenly throughout China's vast land mass. Thus with the new structure of primary education, the Chinese seek to develop education as equitably as possible for large masses of people ranging from illiterate in the rural areas (with 85% of the population) to the more fortunate groups with more opportunity for education in urban centers (15% of the population). This also is the objective in implementing the new national policy of compulsory education for grades 1-9. Compulsory education for all Chinese youth for grades 10-12 in the rural areas is, perhaps, the next step in educational progress.

Grades 7-9 make up junior middle school. Students' alternatives after junior middle are entrance to senior middle school (grades 10-12), a technical/ agricultural school or being assigned to work in factories or on farms. We were told that in rural areas, 70% of the students do not continue their education beyond junior middle school; but in the urban areas, 100% continue in school, with 40% going to senior academic schools and 60% to technical schools. Upon completion of grade 12, a student may take the university entrance examination on which decisions about further education are based. In 1984, 26% of 1,640,000 applicants were admitted to university study.

Perhaps we should insert a note here regarding the courses termed "Politics" in Chinese schools. We did not have sufficient time to probe this aspect of China's educational system. But like our own society, China's is informed by a system of values. These values play a large role in determining where and how social resources are allocated. In China's case, these values are manifested in an ideology based on the ideas of Marx, Engels, Lenin, and

Mao. We presume that courses in politics taught in the schools deal with this ideology and its implications.

As part of the drive to recover from the crippling effect of the Cultural Revolution and to achieve modernization, some primary and middle schools as well as universities/institutes were designated "key" institutions. Most of these are located in urban areas. In short, these schools are university prep schools designed to serve the education of talented students. They receive a larger allocation of resources and are staffed by master teachers. Admission is based on performance on rigorous examinations and about 10% of students are accepted. The curriculum offers comprehensive study with special emphasis on Mathematics, Chinese Language, the Sciences, English and Politics.

Alternatives to attending an academic senior middle school are the technical/vocational/agricultural schools. These schools emphasize combining theory with practice with about 60% of time spent on academic study and 40% on practical work experience. We visited one such school in a rural area near Guangzhou and, even here, found the academic level to be high. We also visited ordinary primary middle schools and had the same reaction.

Just as primary and middle school has been restructured from 10 (5, 5) to 12 years (6, 6) following the Cultural Revolution, so university study has been lengthened from 3 to 4 years. Bachelors degrees are now offered as are master's degrees. Doctoral programs have been re-instituted very recently.

By action taken by the Sixth National People's Congress, September 10 each year is designated National Teacher's Day. The first one was celebrated in 1985. The purpose of the celebration is to raise the social and political status of teachers in order to make teaching one of the most respected and admired professions in society and to create a social atmosphere of respect for teachers, knowledge, and talent. On September 10, 1986 a huge assembly

was held in Beijing to commend 1000 educators, 100 educational institutions and 100 counties in primary education. Top government officials awarded medals and bonuses for meritorious service and the State Education Commission organized lecture tours for outstanding teachers. Top officials of the Commission also attended meetings of teachers, commemorative postal stamps were issued and a ¥ 1 million fund is being established for use in rewarding outstanding teachers. (Note: $1 = ¥ 3.70)

Primary and Middle Schools

The Classrooms. Classes are large with 45-60 students who sit in pairs facing the teacher. Students are very attentive and often give answers in unison during practice. Students always raise their hands in a very disciplined manner - no hand waving - to be called on by the teacher and stand when responding. It is obvious that classrooms in China are not social centers; rather, they are places for serious learning. We never observed a student leaving a class for any reason, falling asleep, or acting disruptive or disrespectful.

There is a teacher's table in front of the room and blackboards on the front (and sometimes rear) wall. Lighting is adequate (bare neon lightbulbs) and very good natural lighting comes from one or two walls of windows. Walls are plastered and painted white and floors are unpainted concrete. Posters are frequently seen on walls, and sometimes student's performance on tests is recorded on a large hanging poster for all to see. Almost all rooms open to outside corridors or courtyards. Even when it is cold, rooms have little or no heat (stoves in some rooms) and windows are often open. Thus, students and teachers commonly wear layered clothing, jackets and rubber-type boots over shoes.

There are no lockers as is common in our schools. All students come to class prepared, each with a box (containing pencils, eraser, compass, straight edge, protractor), textbooks and examination type booklets for notes, seat work, and homework. Though we saw overhead projectors and tape recorders used, the primary teaching aid is the blackboard with both white and colored chalk used. The board work of teachers is excellent. Teachers frequently use small portable blackboards (e.g., 2' x 1 1/2') on which problems, solutions, or notes are written during preparation periods and which can easily be hung from the top of the main blackboard for all to see. These are very effectively used as part of the lesson presentation. There is always good rapport between teacher and students and both greet each other to begin class and say goodbye at the end. Quite commonly students are called on for board work and are supervised during work at their desks.

But what about "problem kids?" First, there do not seem to be many! In milder cases, students are advised by students (the collective), teachers, or parents when their behavior is inappropriate. Apparently this is enough to handle most such cases. If not, as in the extreme cases, students may be sent to a special adjustment school wherein they receive extra help, advice, or counseling - hopefully enough in order for them to work within the usual guidelines. This is viewed as a short term arrangement before these students are returned to their "home" schools. We were told that in Hangzhou (1.5 million population) there is but one "special adjustment" school with about 300 students. There is no evidence of physical punishment; rather, advice seems to be the main approach in a context in which a strong expectation of good behavior is implicit.

To be sure, Chinese students are kids, and kids will be kids! They push, shove, play and "horse around" - <u>but not in the classes</u>. We never observed

students off-task in class. There is discipline (a national characteristic, we think) and control in the class and freedom and spontaneity outside the class. We note here that dating and marriage have traditionally been postponed until, for example, post-college age. While some college students have begun dating recently, parents and school personnel are "holding the line" at the middle school level.

Student School Day

The school day begins at 7:30 a.m. for most students. Students in need of extra help, counseling, or who assist with custodial work arrive by 7:00. The first period begins about 8:00 a.m. There are then four 45-minute periods up to 11:30 at which time a two-hour lunch period begins. Eye exercises are done by all students for about five minutes every morning. There is also a 15-minute mid-morning physical exercise period in which students go outside enmasse to do calesthenics. Most students return home for lunch. For others, after eating it is a time for rest (naps for primary children) and/or play. Two more periods are scheduled from 1:30 - 3:30. After-school activities occur from 3:30 - 5:00: clubs, music, art, study help, and sports. Homework is common and appropriately named since the parents (the home) may help students or at least see that the work is done. Students retire early as does the whole family. Classes meet six days a week, with four of six periods on Saturday.

Teacher School Day

In urban schools, teachers frequently have two periods of instruction per day and thus teach only 10-12 periods per week (in rural areas 18-24 periods). The rest of their time is spent in preparation for classes, correcting homework, helping students, and developing tests. They also have the usual committee and Party work as well as supervision of after-school activities.

Those in schools with resident students may also have evening homework supervisory responsibilities.

There is another important aspect of teaching in China, namely, the emphasis placed on "renewal" or continuing education. It is common for teachers to meet, say, a half-day each week just to discuss ways of teaching various topics. Teachers also are encouraged (and supported) to engage in further study. This may occur by attending courses offered in their school or in a nearby college or university. Correspondence and television courses offer another means of continuing education. A teacher may also be released for a long period in order to engage in full time or advanced study.

We spent an evening in the home of a principal who also teaches third year senior middle school mathematics. His wife teaches mathematics in a primary school. They live together with a grandmother and son in a three-room apartment of fifty-two square meters area: two bedrooms, a living (sitting) room and a very small kitchen (it's only function is for cooking). This was a "typical" teacher's apartment located on campus or very near to the school, which is also typical.

The principal's salary is presently ¥ 109 per month, which is at the 4th level from the top on a 10-level scale. The 1st level salary is ¥ 150/month and the entry salary (10th level) is ¥ 50/month. The principal determines raises for his teachers each year and, in turn, he is evaluated for a salary increase every two years by the Municipal Education Bureau.

We talked about the life of a mathematics teacher in China. Chinese teachers have a full daytime schedule of professional duties. But, in their "off time," they like to go for walks, listen to music, watch TV, read, just relax. They also have opportunities to go to the opera and theater. Some

teachers who speak foreign languages may work as tour guides when schools are not in session.

The "average" middle school teacher's salary is about ¥ 100/month; housing costs about ¥ 4/month and electricity about ¥ 8/month (considered very expensive). Food is reasonably priced. Education and medical services are free. Teachers and factory workers earn about the same salary but, in contrast, industrial workers can increase their salaries through production bonuses. A male teacher can retire at age 60 and a female at 55 at about 90% of salary. Retirement benefits are provided by the government.

The State Council has given the new State Education Commission power to raise teacher salaries and improve working and living conditions. The State Education Commission has replaced the Ministry of Education and carries much more power under the new alignment.

The Mathematics Curriculum

Primary. The New Unified Series of mathematics textbooks is used exclusively throughout China. These are consumable paperbacks which measure approximately 13 cm x 18 cm. The books are a revision of the first texts published in 1979-80. A major revision is now underway to prepare books consistent with the new K-9 compulsory school requirement. The primary texts make some use of color, diagrams, and drawings. There are 10 texts of about 100 pages for the primary grades.

The content of the books is roughly the content of our K-8 curriculum: four basic operations on whole numbers, fractions, and decimals, with decimals treated before fractions. All four operations are extended to the negative numbers beginning in grade 5. Computation is generally more complicated than what is characteristic of our schools. Percentage, ratio and proportion are taught in grade 5, along with some statistics. Geometry is integrated

throughout the grades with a greater emphasis than in our books. By third and fourth grade students are using compasses and straightedge for constructions and protractors to measure and classify angles. Algebra is emphasized very early, first with missing addends and later by solving equations such as $x + 14 = 27$, $x - 3.2 = 11.7$ and $142 - x = 83$. The introduction is formal and great emphasis is placed on finding the solutions mentally. By grades 4-5, general linear equations are solved as are word problems. Computation problems, including mental, are done in both vertical and horizontal form.

We observed numerous interesting mathematics lessons. In a grade 1 class the teacher used a transparency on an overhead showing a blue pond with green grass surrounding it. Then she stated a problem, as follows: "There are some ducks on the pond. There are two ducks ahead of two ducks. There are two ducks behind two ducks. There are two ducks between two ducks. How many ducks are there altogether?" The children thought about the problem and offered their answers, each of which was discussed. Finally, the children agreed that four ducks in a line is the best solution. Children were encouraged to explain their method of solution and various children were called on in order to consider alternate methods. In the same grade 1 class, the teacher wrote

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

on the board and asked the children to quickly find the sum and explain how they got it (e.g., 1 + 9, 2 + 8, etc.). The theme of that part of the lesson was patterns and many sequences of numbers were examined (e.g., 85 80 75 __, __, __; 12, 23, 34, __, __, __). In another example of grade 1 problem solving the teacher showed a transparency in which only 4 heads (of geese) could be seen, but 12 feet were visible. The teacher asked "How many geese are there?" The problem generated a great deal of discussion, with a few

wrong answers given, but the children came to see that the solution was six geese. In general, the emphasis seemed clearly on process (thinking) as opposed to product (answers). In general, grades 1-3 classes were more informal than 4-6 classes. We also observed teachers dividing the class into small groups. Then the teacher posed problems for the groups to solve. Group scores were kept and a winner was declared, with class recognition.

In a grade 4 class of 54 children (non-key school), we observed a lesson on order of operations and the distributive property. The teacher worked with the class to see that $12 \times 3 - 4 \times 3 = (12 - 4) \times 3$. Then she worked these problems with the class:

1. $99 \times 23 = (100 - 1) \times 23 = 2300 - 23 = 2277$

2. $197 \times 12 = (200 - 3) \times 12$
   $= 2400 - 36$
   $= 2364$

Note: To find 2400 - 36, students

think

$$\begin{array}{r} 3910 \\ 2400 \\ -\ 36 \\ \hline 2364 \end{array}$$

but write

$$\begin{array}{r} .. \\ 2400 \\ -\ 36 \\ \hline 2364 \end{array}$$

The class then practiced on 98 x 17, 11 x 295, and 105 x 19. A complete lesson plan for a grade 5 geometry class is given in Figure 1.

Insert Figure 1 about here

In discussions following classes, teachers described how education has changed. For example, in 1949 and after, children were "force fed" mathematics - "tell the knowledge" to use their words. But in recent years reform has included more discovery activities, games, and discussion. Other subjects included in the primary curriculum are Chinese, Moral Education

(which includes how to be a good student), History, Geography, Natural Science, Art, Physical Culture, and Music. English is taught in many schools, beginning in grade 4. In general, there is no homework in grade 1, but it is common from grade 2 onwards.

In its 1984 visit to China, the US - China People's Friendship Association's delegation reported little or no use of calculators and microcomputers in mathematics teaching (cf. Becker, et al., 1984). Things have changed. Our delegation visited a number of primary schools with microcomputer labs. The main language used is BASIC and the use of LOGO is just emerging at this level. However, we saw no use of calculators even though we saw them on sale in numerous shops and department stores (Casio, Sharp, Texas Instruments).

Middle School. Both ordinary and key middle schools offer fourteen courses: Chinese, Mathematics, Foreign Languages (English being the most common, but also Russian, Japanese, French and German), Physics, Chemistry, Biology, History, Politics, Geography, Health Education, Physical Education, Music, Art, and Labor Skills. As we have indicated, Chinese, mathematics, and foreign languages receive more emphasis, in terms of time, than each of the other subjects. Figure 2 seems to show that "mathematics is basic to the study of all other subjects" and provides a breakdown, by hours of instruction, for each course. Further, we see that senior 2 and 3 students have opportunities for elective study, the most common of which are Computers, Mechanical Drawing, Electrotechnics, Astronomy, Oceonography, Medicine, Classical Chinese, Literature, History, Theory of Literature and Art, and a second foreign language. In addition, Labor Skills training provides a correct attitude towards labor and a working knowledge of certain labor skills. If we add homework, it is abundantly clear that middle school

education is a serious enterprise. Figure 3 provides a brief description of topics in the middle school mathematics curriculum.

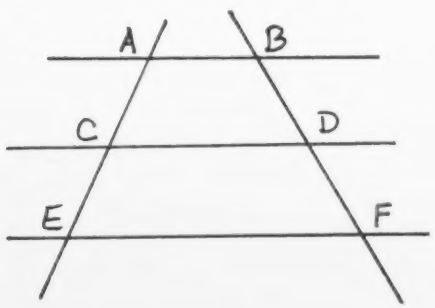
Insert Figures 2 and 3 about here

The overall objective in middle school mathematics is to enable students to gain a knowledge of mathematics considered basic to the study of science and technology, ability to perform quick and accurate operations, and to think logically and use spatial ability as a basis for analysis and solutions of problems. The background in calculus enables 1) students who enter production after graduation to read technical books and continue their own study and, 2) students who go on to the university to study physics with calculus as a base. The background in probability and statistics enables students to use their knowledge both in a practical manner and to continue later, more advanced study.

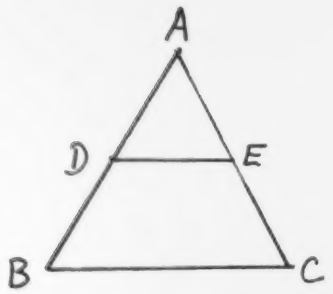
The main topics of middle school mathematics are reviewed in the last semester when students prepare for the university entrance examination. There are seven examinations for middle school graduates who intend to major in mathematics, engineering, sciences, agriculture or medicine: Mathematics, Chinese, English, Politics, Physics, Chemistry, and Biology. For graduates who are interested in social sciences or humanities, six examinations are required: Mathematics, Chinese, English, Politics, History and Geography. Examination questions for Chinese, English, and Politics are the same for both groups, but mathematics problems are given at different levels - those intending to study Science and Technology and those intending to study humanities or "liberal arts."

Our delegation visited numerous middle school classrooms, usually convening afterwards for discussion with the teacher and other staff. In general, middle school classes are more formal than primary ones. Yet, we observed a great deal of interaction between teacher and students in some instances. But the overall student protocol remains the same - be attentive, sit erect, be involved, raise hands to respond, and go to work immediately when told to do so.

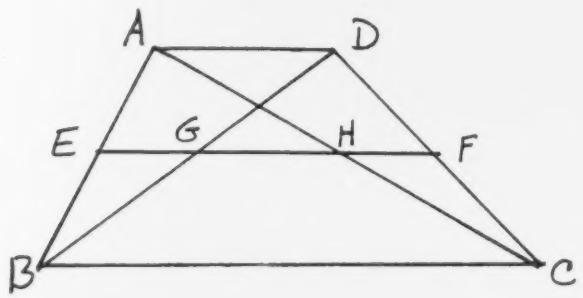
Very frequently we saw the teacher ask a series of questions and then write student responses on the board as they were given, with several or many students joining in. We observed teachers asking both "drill" and "thinking" questions. In a junior middle 2 class (grade 8), the teacher hung a small blackboard from the top of the main blackboard with the following figure:



When the teacher wrote "AB ∥ CD ∥ EF and AC = CE ⇒ ?," the students quickly responded "BD = DF." When she wrote "AB ∥ EF, AC = CE, BD = DF ⇒ ?," the students quickly responded "CD ∥ AB ∥ EF and CD = 1/2 (AB + EF)." Next the teacher hung another small blackboard with the following figure:



The teacher then wrote "AD = DB, DE ∥ BC ⇒ ?" with quick responses "AE = EC." Next, teacher: "AD = DB, AE = EC ⇒ ?" with the quick responses "DE ∥ BC and DE = 1/2 BC." Next, the teacher hung a small blackboard with the following figure:



teacher: "AD ∥ BC, E, F are midpoints of AB and DC, respectively, and EG = GH = HF. $\frac{AD}{BC}$ = ?" There followed two more somewhat more complicated figures and questions on which students worked at their desks.

The teacher was full of excitement with a smiling face as she led students into the work. She walked the aisles observing students during seatwork. We thought this lesson represented a nice example of logical reasoning from given information. The class, in general, was a mix of lecture and interaction.

We also observed a senior middle 2 (grade 11) lesson in analytic geometry. The lesson lasted 45 minutes and is summarized in Figure 4. The

problems are numbers 6 and 8 of the problem set shown in Figure 5. The reader will note that the problems are difficult and are typical of the other problems in the set which students had in hand.

Insert Figures 4 and 5 about here

This class was one of the more formal ones we observed. The teacher was very much "in command" and set up the problems by asking questions and writing student responses on the blackboard until the solutions were complete. This teacher collects student notebooks every 3-4 days in which are written both the problems and their solutions. We examined a few notebooks and noticed that comments and corrections and the date he examined the work were written in red pencil. The teacher faced the blackboard much of the time and gave a casual wave of his right hand when a student completed a response to his question - we thought the practice a bit "off handed" but students did not seem to mind.

In discussion with mathematics teachers after this class, we were told that these problems are on the same level of difficulty as those of the university entrance examinations. Further, the teacher told us that about 50-65% of his students handle this work well, others need some help, but that only 5-10% need "more help and further work." We were also told that the math department chairman prepares the schedule for each teacher and approves his teachers' lesson plans ahead of time. His role also includes guidance of the younger teachers.

Some of the "older" teachers in this school still have no Bachelor's degree and, therefore, are studying in the university while teaching. Some teachers spend three hours each Tuesday afternoon preparing students for

mathematics contests, some of whom have won prizes for the school. The areas in which they work are inequalities, plane geometry, graph theory, solid geometry, analytic geometry, and number theory. All students have access to teachers for extra help during teachers' out-of-class time.

There are always surprises when visiting Chinese schools. Can you imagine American secondary students translating, say, Chinese, German, or Russian mathematics materials into English? We met a teacher in Shanghai who teaches an "English Course Specialized In Mathematics" as an "optional" course in a middle school mathematics department. Emphasis is placed on reading and translation. Our delegation was given samples of translations from American college mathematics textbooks into Chinese. The samples were from the areas of Set Theory, Functions, Analytic Geometry, Number Theory, Logic and Trigonometry. Some were also of material from S. K. Stein's book Mathematics: The Man-Made Universe (1977). By any measure, we think this is an impressive activity for it requires of the translator, a middle school student, a high degree of mathematical knowledge as well as excellent facility in English.

Contrary to practices during the Cultural Revolution and immediately thereafter, examinations are now common in China. There are entrance examinations to key schools and colleges/universities and even in ordinary middle schools there are mid-terms and final examinations. Students who pass the final exam at the end of a school year are promoted to the next grade. Final examinations at the end of third year junior and third year senior are regarded as graduation examinations - those who pass can graduate. Those who do not must pass a make-up examination before promotion or graduation. Senior middle examinations are given, as a matter of policy, in Chinese, Mathematics,

Foreign Languages, Physics, Chemistry, and Politics. Beyond this, teachers also make up and administer chapter tests.

Moral Education is emphasized in the middle schools as a matter of educational policy. But it begins earlier in the primary schools where students learn proper behavior and a "proper political orientation." If young students give a good account of themselves and show the proper physical, intellectual, and moral development, they may become members of the Young Pioneers, an organization of younger students. Such students are easily identified by their red neckerchief (a subset of our delegation were made "honorary" Young Pioneers during a visit to an ordinary primary school in Guangzhou). The objective, of course, is for all children to aspire to become Young Pioneers, though not all make it. But those that do have their own activities regularly and become leaders in the middle school.

At the middle school level, students are taught Socialist morals and learn the basics of Marxism. They are also taught to study hard, show concern for their health, and to observe the "Rules for Middle School Students" (see Figure 6). Patriotism and the importance of the group, as opposed to the individual, are also emphasized. The middle school analogue to the Young Pioneers is the Youth League, grassroots units of which exist in all middle schools. They help the Party organization to educate members in Socialist ideals. Any student of age fourteen or older can aspire to membership, and members wear an identifying badge and are frequently seen as leaders in their classes. Finally, the Student Union is an organization of middle school students which works with the school administration to schedule cultural, recreational, and athletic activities. The leaders of the Union are elected by fellow students for one-year terms.

Insert Figure 6 about here

Children's Palaces

On a Sunday morning in Shanghai the delegation visited a Children's Palace where a wide variety of enrichment activities are provided for gifted and talented children of ages three on up to middle school. Here the delegation saw children being entertained by a magician and children practicing ballet, piano, classical Chinese musical instruments, chorus, shipbuilding, model airplane construction, accordions, gymnastics, electronics, computer programming, science experiments and a host of other activities. Our "guides" during this visit were from a class of 10-11 year old students of English.

Students are recommended for attendance by their teachers in school who feel they have high potential in particular areas. The classes are offered for minimal tuition during after-school hours and on Sundays. This particular Palace is for the most talented in Shanghai, which we could readily believe after seeing them perform. In the Fine Arts building girls of ages 9-12 were practicing ballet and acrobatics with proficient detail. One young girl performed The Swan for us and we were told she performed for President Reagan at the White House in 1984. Other impromptu performances were given for us by students on the piano (ages 3-7), the accordion (ages 7-10) and various Chinese traditional and classical stringed instruments (ages 7-10). Children could be seen independently practicing in hallways.

After observing ballet and various musical groups, to our happy surprise, we found ourselves in what appeared to be a mathematics classroom. This turned out to be a fairly large group of junior middle school students solving

problems, led by a mathematics teacher. The problems were from the 1986 Nationwide Junior High School Mathematics Competition (see Figure 7). In other rooms activities in chemistry, physics, and biology were underway. There was also an aeronautics laboratory complete with model planes.

Insert Figure 7 about here

Later our group was in an excellent microcomputer lab, which had what appeared to be Apple IIe's. Here were students working on a variety of problems, programming in LOGO, BASIC, and Pascal. While all projects were interesting, there was one which quickly caught our eye. The student was developing a graphical model of physical relationships; in this case he was exploring how the water level in a bathtub varies before, during, and after a bath. The student had succeeded in writing a BASIC program (computer graphics) which showed the following: 1) on the top of the screen was a graphic of water flowing into a tub from a faucet, a person going through the process of stepping into the tub and taking a bath, and the water level rising and falling or remaining constant, and 2) on the bottom of the screen was a graphical model of what was happening in 1. In the end, the person in 1 was out of the tub and the full model in 2 was displayed on the screen. (It reminded us of the beautiful mathematical modeling curriculum materials that are being developed at the Shell Mathematics Education Research Centre at Nottingham University in England.)

In the Computer Lab we met the 1986 winner of the Chinese National Computer Competition. He spoke with us, as did a member of the 1986 Chinese International Mathematics Olympiad (IMO) team which, in its first year of participation, placed third in the competition in Warsaw, Poland (the U.S. and U.S.S.R. had tied for first place). Both spoke excellent English and were

obviously superb young scholars. We comment, parenthetically, that the U.S. Annual High School Mathematics Examination (AHSME) has been given the last two years in Beijing, Shanghai, and Tianjin with results that surpased both the U.S. and United Kingdom. That China is placing great emphasis on mathematical contests (at the municipal, provincial, national, and international levels) is very clear, for we had discussed this topic at several middle schools and the Shanghai Hall of Science and now we had observed another aspect of this in the Children's Palace. Mathematics competitions are also being introduced for primary school students in Beijing, with leadership coming from Beijing Teacher's College.

Teacher Preparation

Primary School Pre-Service Training. Teacher education has recently undergone significant change with great emphasis placed on both pre- and inservice preparation of teachers. During the Cultural Revolution major emphasis was placed on the political attitudes rather than academic training of teachers. This policy resulted in an abundance of inadequately trained classroom teachers. The problem was a serious one in urban schools, but a particularly acute one in rural areas. Hence, the great emphasis now on mathematics in preservice programs and in renewal or continuing education.

Unlike our American tradition, the education of primary teachers begins at about age 15! Junior middle school students who aspire to primary teaching must take an entrance examination for Normal School, the name given to institutions preparing primary teachers. If admitted at the end of lower middle school (grade 9), the student then enters a normal school for four years of training. During this period the student completes roughly the equivalent of the last three years of the senior middle school, in addition to a Theory of Arithmetic course, a "methods" course, and other "methods" training for

primary school subjects. The student will also typically spend 2-3 weeks observing classes in nearby schools and later, about six weeks of student teaching. The first year of teaching is spent as an apprentice under close supervision of a senior teacher.

In general, primary teachers teach all subjects, especially in the rural areas. However, it is common for specialist teachers to handle mathematics in grades 4-6, and this is especially true in key schools. The number of normal schools (sometimes called technical schools) has grown steadily in the last few years in order to improve the elementary education of students.

Four years of education in a Normal School is probably the ideal, for one meets teachers who have had three, or sometimes only two, years of normal school training. But these same teachers are usually engaged in continuing education to formally complete their training. In many cases such teachers receive time off to attend classes with their expenses paid. Certainly a tradition has now been established to get a solid background of education before entering the classroom.

Middle School Pre-Service Training. Under ideal conditions, preservice education of middle school mathematics teachers begins after graduation from a middle school and satisfactory performance on the university entrance examination. Students enter a Normal University, which draws and places students nationally, or a Normal College, which draws and places students locally. It should be noted that these institutions are frequently scientific universities as opposed to "teacher training colleges." Preservice teachers study alongside regular students of mathematics, taking the same courses and with the same expectations. The course of study is very rigorous and the vast majority of courses are mathematics. Figure 8 shows the course of study for

middle school mathematics teacher training in Beijing Normal University, which is a key university. Note the heavy emphasis on mathematics.

China has no counterpart to our professional education sequence of course nor to our liberal arts requirements. To be sure, there is a "methods" course and student teaching. But beyond that, students' time is pretty much spent in strictly academic work: 63% - mathematics, 12% - foreign language, 6% - physics, 6% - mathematics "methods" and education, and 12% - psychology, philosophy, economics, history (which comprise political studies). (Note: practice teaching is not included in these percentages.) Typically the "methods" course is concerned with methods of teaching algebra and geometry and making and using teaching materials. Readings in cognitive psychology may also be included. Noticeably absent is any reference to remedial courses so characteristic of our own colleges/universities. All courses are taught in the mathematics department by its staff or by instructors from outside the department who come to teach courses for the mathematics department. At Yunnan Province Normal University, not a key university, an almost identical program of studies is required (cf., Becker, 1984). The program of studies leads to a Bachelor's degree and there is no formal counterpart to our certification process.

The program outlined in Figure 8 is the ideal, as mentioned. In fact, Chinese mathematics educators told us they have a serious problem of inadequately trained middle school mathematics teachers (though members of our delegation wondered about their meaning of the term "inadequate"). During the Cultural Revolution with its emphasis on political attitudes, at the expense of academic training, apparently many students who completed senior middle school were assigned to teach lower middle courses, and students who only

completed lower middle were assigned to teach primary school courses. Thus, we see the importance of inservice training now - to raise the level of mathematical competence among teachers who entered the profession during the Cultural Revolution.

Insert Figure 8 about here

The delegation visited the Hangzhou College of Education. Here was found an extensive inservice program for middle school mathematics teachers. In fact, the college deals almost exclusively with inservice programs (full or parttime). Other approaches to inservice include correspondence and TV courses. Incentives are given to teachers in the form of tuition waivers and salary increases. Many teachers attend 3-5 years to upgrade their backgrounds.

The government is experimenting with other approaches to handling the shortage of good teachers. It is a problem which will become more critical in the future since a large number of middle school teachers are nearing retirement and it may be difficult to replace them by younger teachers. As an incentive to enter teaching, the government offers an opportunity to senior middle graduates to take the university entrance examination early. Thus, if they do not pass, they can retake it at the regularly scheduled time which still permits fall admission. In other cases, a greater opportunity to get an apartment serves as an incentive. In another development, the State Education Commission is offering stipends or loans (not to exceed ¥ 400) to students who go into mathematics teaching, with a stipulation that the loans need not be repaid if the student agrees to teach for at least five years or to teach in a remote area. Since there is an attitude that to teach in the schools is to

make a sacrifice in salary, the State Education Commission is also working out schemes for increasing teacher salaries.

Summary and Conclusions

Our delegation travelled in China for nearly three weeks. During this time we visited many schools and observed instruction in numerous classrooms at all levels. Though most of the schools we visited were key schools, we sense that many of our observations might very well be characteristic of ordinary schools too. Unfortunately, we spent very little time in rural areas and no time at all in areas that would be considered remote.

Some of our visits provided time to discuss what we had seen and others were more hurried than we would have liked. All this is understandable since the demands of our schedule were substantial. Of necessity, we also had other diplomatic and social functions in addition to the professional activities. As immensely nice and enjoyable as the other functions were, they took time from our professional schedule. Then, too, we communicated through the medium of translation which, all things considered, reduced the speed and accuracy of the information flow even though the translators were excellent. At times, plain and simple fatigue took its toll.

Even so, there is no doubt in our minds that the Chinese have made great progress in implementing educational reform, that their students are learning a great deal of mathematics, and that, while doing so, they enjoy school and life a great deal. There is also little doubt that at any grade level, the students we observed seem ahead of our own in terms of both topics studied and mastery thereof. Since none of us believes that Chinese youth are innately more intelligent than our own, possible explanations lie elsewhere. These we mention briefly below:

1. Chinese students try harder. They appear to study for long hours and have fewer social/recreational diversions. Their curriculum is more intensive and their teachers seem to give no more, and maybe less, homework than our own.
2. They spend more time in mathematics classes - we estimate 10-15% more time, not taking enrichment classes into consideration in either case.
3. Parents, teachers, and society in general expect more, so naturally there is a tendency to produce more. More than a few of our group have commented that they will now raise their expectations of students in their classes and let the students know it.
4. A healthy mind in a healthy body . . . this is something seen emphasized everywhere we visited. In contrast to Chinese students, our own seemingly carry the burdens of modern society with them into classrooms. We refer to drugs, alcohol, and the likes of over indulgence in sugar and fast foods, not to mention the breakdown of the traditional family, single-parenting, latch-key children and the anguish of dealing with too many options. We also have great societal emphasis on style and appearance, popularity, money and romance. These would seem to interfere with establishing a sound, basic confidence about oneself which plays a vital role in learning.
5. Chinese teachers feel more positive about their jobs and their role in society. This may also play a role in students' learning. This is not to say that their teachers

> do not experience the frustrations of our own, for they speak about the need for better salaries, living conditions, and status. Still, on balance, we sense that they feel more satisfied than our teachers.

China did not participate in either the First or Second International Mathematics Studies (FIMS, SIMS). But if it had, we wonder whether achievement data may have shown results consistent with that of the Japanese in comparison to the United States. Horvath (1987) writes that the results of the FIMS and SIMS studies confirmed what she had earlier observed (emphasis added) in Japanese classrooms and had concluded from interviews with students, teachers, and counselors (p. 367). Like Horvath, we feel "that more than minor adjustments are needed if the level of American junior and senior high school mathematics achievement is to improve" (p. 367).

We have a strong feeling that if the current educational policies remain in place and newly proposed ones are implemented, China will have a large number of highly and excellently trained young people who will make possible the realization of it's drive towards scientific/technological development and modernization. Some twenty-five years ago there was great concern in our country about "catching up" with the Soviet Union by improving our mathematics and science programs. Presently we are amazed at the technology and overall mathematical literacy of the Japanese. We wonder whether we may be pondering the great progress made by the Chinese in another twenty-five years. Finally, Davis and Romberg (1979) have commented that:

> There is little to be gained, and much to be lost, by comparing national systems in an "our-system-can-beat-your-system" frame of mind. The important gains are to be had only if each nation tries to learn from the other. . . .One road toward (that) improvement

> may be the careful study of what other nations do, in the hope of learning from them. (p. 29)

Perhaps we can learn some lessons by studying the practices in mathematics education in China.

Acknowledgements

Our trip to China was indeed a Great Adventure. But it was possible only through the efforts of many people. In particular, we wish to thank Ms. Martha Brown (USCPFA) who provided an excellent orientation in Chicago before departure. We also thank the Chinese Mathematical Society (CMS) for its gracious invitation, the China International Conference Center for Science and Technology (CICCST) for handling all arrangements, and the China Association for Science and Technology (CAST) whose Beijing and provincial offices welcomed us on each leg of our journey, as well as all our inquiries and requests. In particular, we wish to thank Zhou Peiyuan, Honorary President of CAST, (Ms.) Wu Ganmei, Director of CICCST, and Yang Lo, Secretary General of CMS. To Ms. Wang Hsin, the National Project Coordinator who travelled with us and looked after us, and to all the Local Project Coordinators goes our heartfelt appreciation. We also thank the US - China People's Friendship Association (USCPFA)/Washington, D.C. and B and A Travel/Carbondale, Illinois for handling all our travel arrangements in an expert and professional manner.

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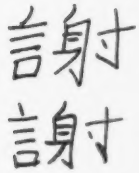
- CASIO, Inc. (29 calculators, mostly scientific)
- Creative Publications (books, teaching aids)
- Cuisenaire Company of America, Inc. (books, teaching aids)
- Dale Seymour Publications (books, teaching aids)
- National Council of Teachers of Mathematics (numerous yearbooks, pamphlets, reports and brochures)
- University of Chicago School Mathematics Project (multiple copies of Transition Mathematics, Algebra I, primary, and calculator materials)
- McDougal, Littell and Co. (geometry textbooks)
- Scott, Foresman and Company (numerous textbooks)
- Addison-Wesley Publishing Company (numerous textbooks)
- Houghton-Mifflin Company (numerous textbooks)
- Laidlaw Educational Publishers (numerous textbooks)
- Open Court Publishing Company (numerous textbooks and other materials)
- Charles E. Merrill Publishing Company (numerous textbooks)
- Lawrence Erlbaum Publishers (authoritative books on problem solving research/thinking skills)
- Computer Science Press (numerous textbooks)
- Mathematical Association of America (mathematics contest materials)
- SUNBURST Communications, Inc. (loan of computer software)

Finally we express personal and professional appreciation to our Chinese friends and colleagues in mathematics education, too numerous to list, who generously invited us into their classrooms and homes and patiently and skillfully answered and asked questions that promoted scholarly exchange, communication,

and friendship. To Mr. Zhou Yiyun goes our thanks for assistance in translating two lesson plans as well as other translation assistance.



("thank you")

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Figure 1

Beijing No. 1 Experimental Primary School

Grade 5 of Primary School/Mathematics Class

Objectives:

1. Help students to understand the property that "The sum of every two sides of a triangle is greater than the third side."

2. Help students to understand the method of constructing a triangle, using ruler and compass, given the lengths of three sides.

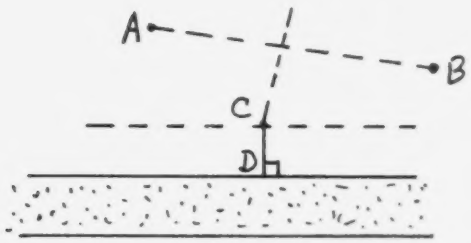
Teaching Time:

About 50 minutes

Teaching Plan:

1. Review:

   A. Ask a question, let students find the solution.



As in the figure, in the north of a river, there are two towns A and B. We want to construct a store which is 30 meters from the north bank of the river and has the same distance to the two towns. Find the position of the store and choose the nearest point to the store.

First, let students find the method to the solution.

   B. Let students answer what other specific figure is formed by the set of points learned before. Give them some examples.

2. Teaching New Content:

   A. Given the lengths of three sides, construct a triangle using ruler and compass.

      1) Given the lengths of three sides of ΔABC: a = BC = 4 cm, b = AB = 2 cm, c = AC = 3 cm. Let students consider how to do the construction.

      2) Let some students explain their methods. Afterwards, the teacher shows the construction on the blackboard: First construct side BC, then lead students to consider that if we can find the position of point A, we can construct the

triangle. However, point A should satisfy two conditions: First, A is 2 cm from B and, second, A is 3 cm from C. According to the method shown in the review part, find the position of point A. If students can find the method and the solution, examine the results together, ask students their reasons.

3) Summarize the method of construction.

4) Give some exercises:

   a) Construct triangle ABC: a = 2.5 cm, b = 4 cm, c = 3.2 cm.

   b) Construct a triangle ABC: a = 5 cm, b = 0.03 m and c = 38 mm.

B. The relationship among the three sides of a triangle.

1) Ask students to construct a triangle: a = 5 cm, b = 1.8 cm, c = 2.2 cm. When the triangle cannot be formed, students will be confused.

2) From this, indicate that it is not true that a triangle can be constructed using any three lengths for sides. Taking 1 and 2 together, lead students to observe the relationship among b, c, and a. Find the necessary condition to construct a triangle given the lengths of the three sides.

3) Ask students if the sum of b and c is less than a, we cannot get point A.

4) To construct ΔABC: a = 5 cm, b = 3 cm, c = 2 cm. Students will see that when b + c = a, they cannot be successful

Definitely indicate it must be b + c > a.

5) Can we say in other words that given three sides of a triangle, the sum of two sides must be greater than the third side?

Give students some examples, let them judge the results:

a = 3 cm, b = 5 cm, c = 1 cm

a = 4 cm, b = 3 cm, c = 1 cm

a = 5 cm, b = 4 cm, c = 8 cm

Summary: Emphasize that in order to form a triangle, the sum of any two sides must be greater than the third side.

6) Conversely, when a triangle is formed, the sum of any two sides must be greater than the third side.

Lead students to consider: Can we sum up the property of a triangle to knowledge we learned before (i.e., the shortest distance between two points is the segment which connects these points)? If students cannot see this result, give them an example: "A student goes to school. There are two ways which can be chosen by the child (as in the figure). Which one is shortest? Why?

Finally, emphasize the statement "The sum of any two sides must be greater than the third side" - the word "any" is necessary.

C. Exercises:

1) Can the lengths of three sides of a triangle be 20 mm, 3 cm, 0.4 dm, respectively? Why? If 3m, 2m, 1 m? Why?

2) Given the lengths of three sides of a triangle (for example, question 1), are(is) there some simple method to determine whether they form a triangle or not? (not necessary to compare three times)

3) If the length of the base of an isosceles triangle is 3 cm and the length of a side is 4 cm, can a triangle be formed? Are there some special limits for constructing an isosceles triangle?

4) Given the length of a side, how do you draw an equilateral triangle? Are there some limits for constructing an equilateral triangle?

3. Summary:

The students and teacher summarize the conclusion together:

A. If you know the lengths of three sides of a triangle, how to construct the triangle.

B. The relationship among the three sides of a triangle.

4. Arrange Homework:

Consider if in a triangle a = BC = 5 cm, b = AB = 3 cm, then what is the longest limit and the shortest limit for c?

Figure 2

The Current Chinese Secondary School Curriculum

Subject	Grade: Junior Middle School 1	2	3	Senior Middle School 1	2	3	Total Hours of Instruction
Political Studies	2	2	2	2	2	2	384
Chinese	6	6	6	5	4	4	1,000
Mathematics	5	6	6	5	5	5	1,026
Foreign Language	5	5	5	5	5	4	932
Physics		2	3	4	3	4	500
Chemistry			3	3	3	3	372
History	3	2		3			266
Geography	3	2			2		234
Biology	2	2				2	192
Health Education			2				64
Physical Education	2	2	2	2	2	2	384
Music	1	1	1				100
Art	1	1	1				100
Hours of Required Courses Per Week	30	31	31	29	26	26	5,554
Electives					4	4	240
Labor Skills	2 weeks*			4 weeks**			

*Four hours a day.
**Six hours a day.

From Secondary Education in China, State Education Commission, PRC, 1986, p. 23.

Figure 3

Description of Topics in the Junior and Senior
Middle School Mathematics Curriculum
(Yu Yuanqing and Cao Feiyu, 1982)

Junior Middle School

Year 7: Rational numbers and their operations; addition and subtraction of polynomials; first degree equations and inequalities in one variable; systems of first degree equations; multiplication and division of polynomials; factorization, fractions and fraction equations.

Year 8: Straight lines, intersecting and parallel lines; triangles; quadrilaterals, extracting root and quadratic radicals, quadratic equations in one variable; exponents and common logarithms; similar figures.

Year 9: Rectangular coordinate system in the plane; solutions of triangles; circles; functions and graphs: Direct and inverse variation, linear functions and quadratic functions; equation of a line and circle; elementary statistics: Mean, mean-squared deviation, standard deviation, and frequency distribution.

Senior Middle School

Year 10: Sets; correspondence; functions; power, exponential, and logarithmic functions; trigonometric functions (graphs/properties); sum and difference identities; multiple and half angle formulas; inverse trigonometric functions; simple trigonometric equations; solid figures: Basic properties of the plane, relation of position between two lines, between a line and plane, two planes; surface areas and volumes of prisms, pyramids, cylinders, cones, frustrums, and spheres; quadratic curves: Circle, ellipse, parabola, hyperbola; translation and rotation of axes; polar coordinates and parametric equations; Spirals of Archimedes; involute; cycloid.

Year 11: Systems of linear equations (determinants of second and third order); inequalities and proofs; complex numbers: Concept, operations; trigonometric form of a complex number; permutations and combinations; mathematical induction and binomial theorem; probability: Equally likely, mutually exclusive, and independent events; composite experiments; plane analytic geometry.

Year 12: Sequences and limits; the derivative and the differential and applications; the indefinite integral and applications; review of senior middle school mathematics.

Figure 4

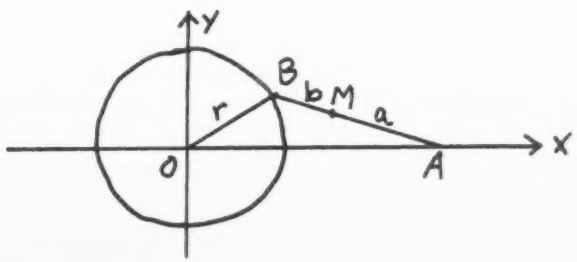
Lesson in Analytic Geometry

Second Year Senior Middle School

Background:

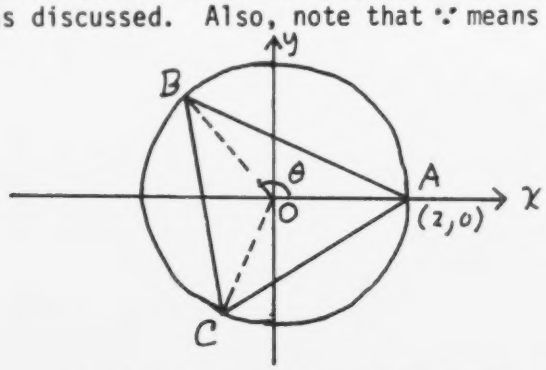
The teacher was continuing work of the previous class. He had written each of the following two problems on small blackboards (2' x 2 1/2') which were hung from the top of the main blackboard for all to easily see. There were 50 students in the class. The students had about 15 minutes to work on the problems, after which the teacher took 20 minutes to work through each of them completely. Only the solution for the second problem is shown.

1. In the following figure, let M be a point on segment BA and MA = a, MB = b. When point A is moving back and forth on line OX, point B rotates around the fixed point O on the circle. Find the locus parametric equation of point M.



2. On the circle $x^2 + y^2 = 4$ there is a fixed point A (2, 0) and two moving points B and C. When $\angle BAC = \frac{\pi}{3}$ (a constant), find the locus equation of the centroid of ΔABC.

Note: The little blackboard showed the figure below without the dotted lines and θ. These were added as the statement of the problem was discussed. Also, note that $\because$ means "if" and $\therefore$ means "then."



Solution:

Let point P, the centroid, be (x, y) and $\angle AOB = \theta$. The coordinates for point B are $(2\cos\theta, 2\sin\theta)$.

$\because \angle BAC = \frac{\pi}{3}$, $\therefore \angle BOC = \frac{2\pi}{3}$.

Therefore **C** is given by $(2\cos(\theta + \frac{2\pi}{3}),\ 2\sin(\theta + \frac{2\pi}{3}))$.

Since P is the centroid of ΔABC,

$$X = \frac{2 + 2\cos\theta + 2\cos(\theta + \frac{2\pi}{3})}{3}$$

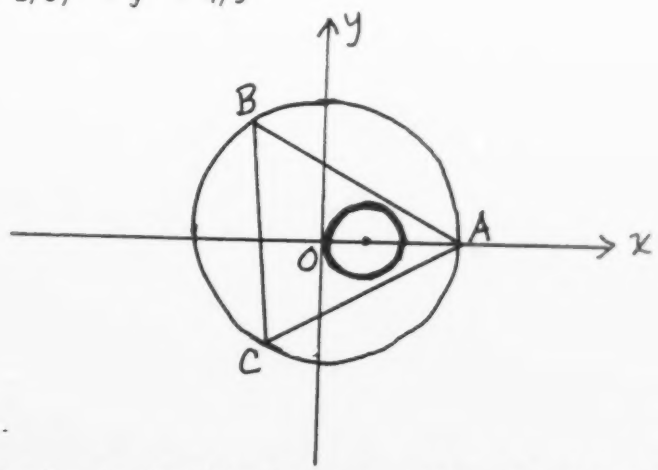
$$Y = \frac{2\sin\theta + 2\sin(\theta + \frac{2\pi}{3})}{3}$$

$$\begin{aligned} X - \tfrac{2}{3} &= \tfrac{2}{3}(\cos\theta + \cos(\theta + \tfrac{2\pi}{3})) \\ &= \tfrac{2}{3}(2\cos\tfrac{\theta+\theta+2\pi/3}{2}\ \cos\tfrac{\theta-\theta-2\pi/3}{2}) \\ &= \tfrac{2}{3}(2\cos(\theta + \pi/3)\ \cos(-\tfrac{\pi}{3})) \\ &= \tfrac{2}{3}\cos(\theta + \pi/3) \end{aligned}$$

$$\begin{aligned} Y &= \tfrac{2}{3}(\sin\theta + \sin(\theta + 2\pi/3)) \\ &= \tfrac{2}{3}(2\sin(\tfrac{\theta+\theta+2\pi/3}{2})\cos(\tfrac{\theta-\theta-2\pi/3}{2})) \\ &= \tfrac{2}{3}(2\sin(\theta + \pi/3)\cos(-\pi/3)) \\ &= \tfrac{2}{3}\sin(\theta + \tfrac{\pi}{3}) \end{aligned}$$

Now, $(x - 2/3)^2 + y^2 = (2/3)^2(\cos^2(\theta + \frac{\pi}{3}) + \sin^2(\theta + \frac{\pi}{3}))$

So, $(x - 2/3)^2 + y^2 = 4/9$



The conclusion is that the locus equation of point P is $(x - 2/3)^2 + y^2 = 4/9$, a circle whose radius is 2/3 and whose center is at (2/3, 0).

Figure 5

Problem Set for Analytic Geometry Class

Second Year Senior Middle School

Exercises on Parametric Equations

1. Transform the parametric equation $\begin{cases} x = \sin\theta \\ y = \cos 2\theta \end{cases}$ (θ is a parameter) into a common equation and draw the graph of the equation on the coordinate system.

2. The parametric equation $\begin{cases} x = 2\tan\alpha \\ y = 1 + 3\sec\alpha \end{cases}$ (α is a parameter) implies its common equation is _____, its vertex is _____, and its focal points are _____. What kind of figure is the graph? _____

3. The coordinates of point M are two roots of equation $t^2 - 2\sqrt{6}\sin(\alpha + \frac{\pi}{3})t + 3\sqrt{3}\sin 2\alpha = 0$.

   A. if α is a parameter, the parametric equation of the locus of point M is _____.

   B. the common equation of the locus of point M is _____.

   C. the graph of the locus of point M is _____.

4. If the parametric equation of the curve is

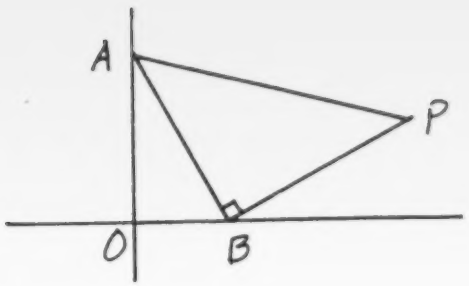
$$x = \frac{1}{2}(e^t + e^{-t})\cos\theta$$
$$y = \frac{1}{2}(e^t - e^{-t})\sin\theta$$ and

   A. if t is constant ($t \neq 0$) and θ is variant, the equation expresses _____.

   B. if t is variant and θ is constant ($\theta \neq \frac{K\pi}{2}$, $K \in N$), the equation expresses _____.

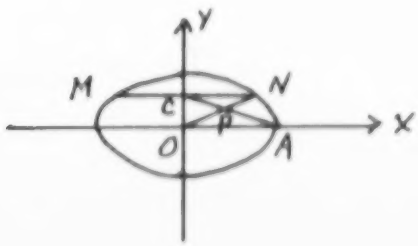
   C. discuss the property of the equations in the two cases, and find the common points of them.

5. In the following figure, let $\triangle ABP$ be an isosceles right triangle, $\angle ABP = 90$ and $|AB| = |BP| = a$. When points A and B are moving on the x-axis and y-axis, find the locus equation of point P.



6. On a circle $x^2 + y^2 = 4$, there are a fixed point a (2, 0) and two moving points B, C. When $\angle BAC = \frac{\pi}{3}$, find the locus equation of the centroid of ΔABC.

7. In the following figure, in the ellipse $x^2/4 + y^2/2 = 1$, a moving chord MN is parallel to the x-axis and meets the y-axis at point C. The segments ON and AC intersect at point P. Find the locus of point P.



8. In the following figure, let M be a point on segment BA and MA = a, MB = b. When point A is moving back and forth on line OX, point B rotates around the fixed point O on the circle. Find the locus parametric equation of point M.

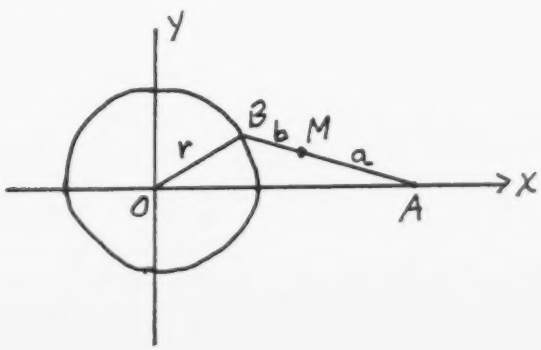


Figure 6

Rules for Middle School Students

1. Love our country and the people, support the leadership of the Chinese Communist Party, study hard and get ready to devote all your abilities to our Socialist modernizations.
2. Be at school on time. Don't be late or leave early or cut classes.
3. Concentrate your mind and think deeply in class; do your homework earnestly.
4. Be active in physical exercises and healthy recreational activities.
5. Take an active part in physical labor and appreciate fruits of labor.
6. Live simply, pay attention to hygiene, don't smoke or drink; don't spit on the floor.
7. Keep the school discipline, observe public orders and obey the State laws.
8. Show respect to teachers, unite with your school mates and be polite to everyone; don't scold or scuffle.
9. Love the collective; take good care of public property and never do anything harmful to the people.
10. Be honest and modest, and be ready to correct your mistakes.

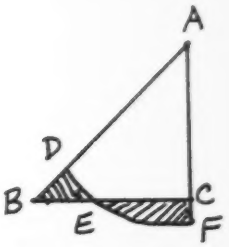
Figure 7

1986 Nationwide Junior High School
Mathematics Competition in the People's Republic of China

8:30 - 10:30 a.m./April 6, 1986

Part I: Place your answer in the blank space following each question: 8 points for each question.

1. If one root of the equation $x^2 + px + q = 0$ is twice as large as the other root, the relationship between p and q is _____.

2. 

In the figure, in a right triangle ABC, AC = BC, the center of the circle on which arc DEF lies is A. If two areas of the shaded parts in the figure are equal, then AD:DB is _____.

3. The natural number N is connected on the right of any natural number. (For example, 2 is connected on the right side of 35, forming a new natural number 352.) If the new natural number formed according to this method can be divided by N, the N is called a "magic number." Among the natural numbers less than 130, how many "magic numbers" are there?

_____

4. If a, b, c are whole numbers and $m = a^2 + b^2$ and $n = c^2 + d^2$, then mn can also be expressed in the form of the sum of two squared whole numbers. mn = _____

5. The parabola $y = -x^2 + 2x + 8$ meets the x-axis at points B and C, and the point D divides the segment BC into two equal parts. Point A is a moving point on the parabola (above the x-axis) and $\angle BAC$ is an acute angle. Then in what range does the length of segment AD vary? _____

Part II: Multiple Choice questions. Place your answer in the brackets following the question. If your answer is correct, you will receive 8 points. If the answer is wrong, you will receive no points. If you do not answer the question, you will get 2 points.

1. If $\log_x a = a$ ($a > 1$ and $a \in N^+$), then x is

A. $10^{a \log_{10} a}$ B. $10^{(\log_{10} a)/a^2}$

C. $10^{(\log_{10} a)/a}$ D. $10^{a \log_{10}(\frac{1}{a})}$

[ ]

2. If $a < b$, then $\sqrt{-(x+a)^3(x+6)}$ is

A. $(x+a)\sqrt{-(x+a)(x+b)}$ B. $(x+a)\sqrt{(x+a)(x+b)}$

C. $-(x+a)\sqrt{-(x+a)(x+b)}$ D. $-(x+a)\sqrt{(x+a)(x+b)}$

[ ]

3. If the equation $||x-2|-1| = a$ has three whole number roots (a is a constant), then a is

A. 0 B. 1

C. 2 D. 3

[ ]

4. $[x]$ expresses the largest whole number which doesn't exceed x, (for example, $[\sqrt{5}] = 2$). Let n be a natural number and

$I = (n+1)^2 + n - [\sqrt{(n+1)^2 + n + 1}]^2$, then

A. $I > 0$

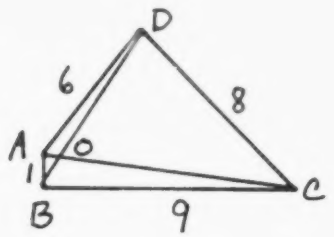
B. $I < 0$

C. $I = 0$

D. when n takes different values, all the three cases A, B, C might occur.

[ ]

5. The lengths of four sides AB, BC, CD, DA of the quadrilateral ABCD are 1, 9, 8, 6 respectively (as in the figure below). Which of the following is true.



A. Quadrilateral ABCD can be circumscribed about a circle.

B. Quadrilateral ABCD cannot be inscribed in a circle.

C. The diagonal lines AC and BD are not perpendicular.

D. $\angle ADC \geq 90$

E. $\triangle$ BCD is an isosceles right triangle

1) (A)T (B)F (D)T

2) (C)T (D)F (E)T

3) (C)T (D)F (E)F

4) (B)F (C)F (D)T

[ ]

END.

Figure 8

Program of Study for Preservice Middle School Teachers (Junior and Senior)

Beijing Normal University (Key University)

Course Title	Hours
Required Courses	
Analytic Geometry	140
Calculus	300
Linear Algebra	120
Abstract Algebra	120
Real Analysis	72
Complex Analysis	72
Probability Theory	72
Ordinary Differential Equations	72
Differential Geometry	72
Methods of Calculation	72
Program Design (BASIC or Pascal)	72
Physics	140
Mathematical Education	72
Education	72
Psychology	72
Philosophy	72
Economics	72
History of China	72
Foreign Language (English, Japanese or Russian)	280
Six - Eight weeks of practice teaching in the middle school.	
Optional Courses	
(The student chooses at least 4 courses from the following list)	
Projective Geometry	72
The Foundations of Geometry	72
Topology	72
Algebraic Topology	72
Partial Differential Equations	72
Theory of Groups	72
Theory of Rings and Fields	72
Cononical Groups	72
Mathematical Logic	72
Theoretical Mechanics	72
Theoretical Statics	72
Stochastic Processes	72
Theory of Numbers	72
Functional Analysis	140
History of Chinese Mathematics	72
Theory of Sets	72